Part I: Answer all of the following multiple choice questions.

- **Do not forget to write your name and mark your student ID number correctly on the Multiple Choice Item Answer Sheet.**
- Mark your MC answers to the boxes in the Multiple Choice Item Answer Sheet provided.
- Mark only one answer for each MC question. Multiple answers entered for each single MC question will result in a 3 point deduction.

Write also your MC question answers in the following boxes for back up use only. The grading will be based on the answers you mark on the MC item answer sheet.

Question	1	2	3	4	5	6	7	8	9	10	11	Total
Answer												

Each of the MC questions except Q1 is worth 5 points. Q1 is worth 2 points. No partial credit.

1. What is the version of your midterm exam paper? (Read the top left corner of the cover page of the exam!)

   (a) Green (b) Orange (c) White (d) Yellow (e) Sample

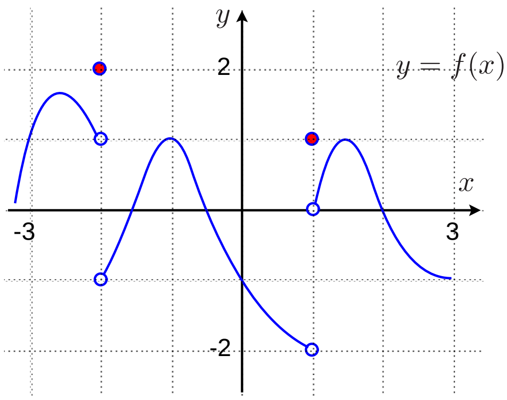
2. Evaluate $\lim_{x\to 2} \frac{x^2+2x-8}{x^2-2x}$

   (a) 1 (b) 2 (c) 3 (d) 4 (e) does not exist

3. Evaluate $\lim_{x\to\infty}(\sqrt{x^2+3x+1}-\sqrt{x^2+x+2})$

(a) 0 (b) 1 (c) 2 (d) 3 (e) ∞

4. Find $\lim_{x\to-2^+} f(f(x))$ where the graph of the function f is given as follows.



(a) 0 (b) 1 (c) -2 (d) -1 (e) Does not exist

5. A function g is defined by

$$g(x) = \begin{cases} 4x - a + 3 & \text{if } x < 1 \\ ax^2 + 3x & \text{if } x \geq 1 \end{cases}$$

where a is a constant. Find a such that g is continuous for all x.

(a) $a = 1$ (b) $a = 2$ (c) $a = 3$ (d) $a = -1$ (e) $a = -2$

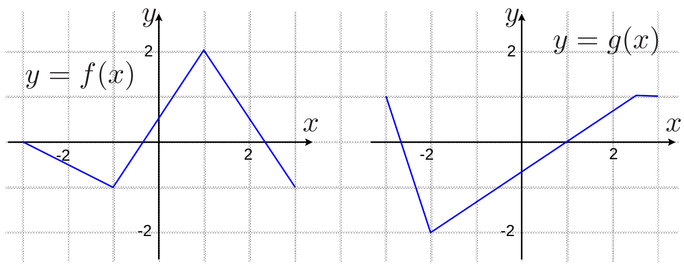
6. Find the slope of the tangent line to the graph of $y = \frac{4}{x^2}$ at the point $(2, 1)$.

(a) $-\frac{1}{2}$ (b) -1 (c) -2 (d) 1 (e) 2

7. Find the derivative $f'(2\pi)$ where $f(x) = \dfrac{x \sin x}{\cos x + 1}$.

(a) π (b) 2π (c) 3π (d) 4π (e) 5π

8. Find the derivative $(f \circ g)'(1)$, where the graphs of the functions f and g are given respectively as follows.

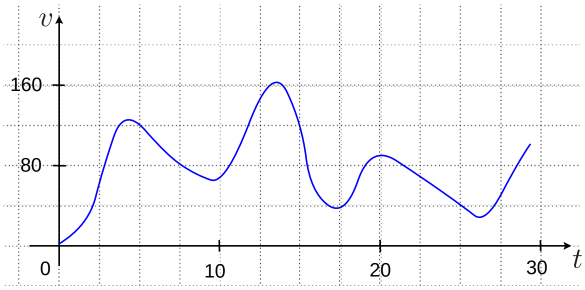


(a) $\frac{1}{2}$ (b) 1 (c) $\frac{2}{3}$ (d) $\frac{3}{2}$ (e) 2

9. $y = f(x) = x^3 + 1$ is a one to one function. If $y = h(x)$ is the inverse function of f, find $h'(2)$.

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{3}{2}$ (d) 2 (e) 3

10. The graph of the velocity function $v = v(t)$ of a runner, in meters per minute, is shown below. At which of the following times is the runner slowing down most rapidly than the other given moments?



(a) $t = 5$ (b) $t = 10$ (c) $t = 15$ (d) $t = 20$ (e) $t = 25$

11. The volume V of a sphere is an increasing function of its area A. What is the rate of change of V with respect to A when $A = 36$ cm^2 ?

(a) $\frac{2}{\pi}$ (b) $\frac{3}{2\sqrt{\pi}}$ (c) $\frac{\sqrt{\pi}}{\sqrt{6}}$ (d) $\frac{3}{\sqrt{\pi}}$ (e) $\frac{2}{3}\pi$

Part II: Answer each of the following questions.

12. ([*14 pts*]) Let $y = f(x)$ be a differentiable function of x.

(a) Express the derivative function $f'(x)$ as a limit. [4 pts]

$$f'(x) \;=\; \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

(b) For the particular function $f(x) = \dfrac{1}{\sqrt{x+1}}$, express $f'(x)$ as a limit. [4 pts]

$$f'(x) \;=\; \lim_{h\to 0} \frac{\frac{1}{\sqrt{x+h+1}} - \frac{1}{\sqrt{x+1}}}{h}$$

(c) Find the limit in part (b). Show your work for full credit! [6 pts]

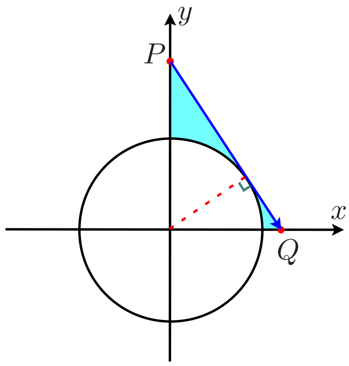
$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{\frac{1}{\sqrt{x+h+1}} - \frac{1}{\sqrt{x+1}}}{h} \\
&= \lim_{h\to 0} \frac{\sqrt{x+1} - \sqrt{x+h+1}}{h\sqrt{x+h+1}\sqrt{x+1}} \\
&= \lim_{h\to 0} \frac{(\sqrt{x+1} - \sqrt{x+h+1})(\sqrt{x+1} + \sqrt{x+h+1})}{h\sqrt{x+h+1}\sqrt{x+1}(\sqrt{x+1} + \sqrt{x+h+1})} \\
&= \lim_{h\to 0} \frac{-h}{h\sqrt{x+h+1}\sqrt{x+1}(\sqrt{x+1} + \sqrt{x+h+1})} \\
&= \lim_{h\to 0} \frac{-1}{\sqrt{x+h+1}\sqrt{x+1}(\sqrt{x+1} - \sqrt{x+h+1})} \\
&= \frac{-1}{2(x+1)^{3/2}}
\end{aligned}$$

13. ([*14 pts*]) A ray emitting from a point P on the positive y-axis above the circle defined by the equation $x^2 + y^2 = 4$ is tangent to the circle in the first quadrant and hits the x-axis at a point Q.

(a) Express the y-coordinate of the point P as a function of the x coordinate of the point Q. (Hint: The tangent is perpendicular to the radius from the center of the circle to the point of contact.) [6 pts]

$$\frac{1}{2}xy = \text{area}(\triangle OPQ) = \frac{1}{2} \cdot \sqrt{x^2 + y^2} \cdot 2$$

$$y = \frac{2x}{\sqrt{x^2 - 4}} \qquad (x > 2)$$



(b) If the point Q is at a distance of $x = 3 + t$ (in meters) from the origin at time $t \geq 0$ (in seconds), How fast is the point P dropping at time $t = 2$? [8 pts]

The y-coordinate of the point P as a function of time is

$$y = \frac{2(3+t)}{\sqrt{(3+t)^2 - 4}}$$

Hence

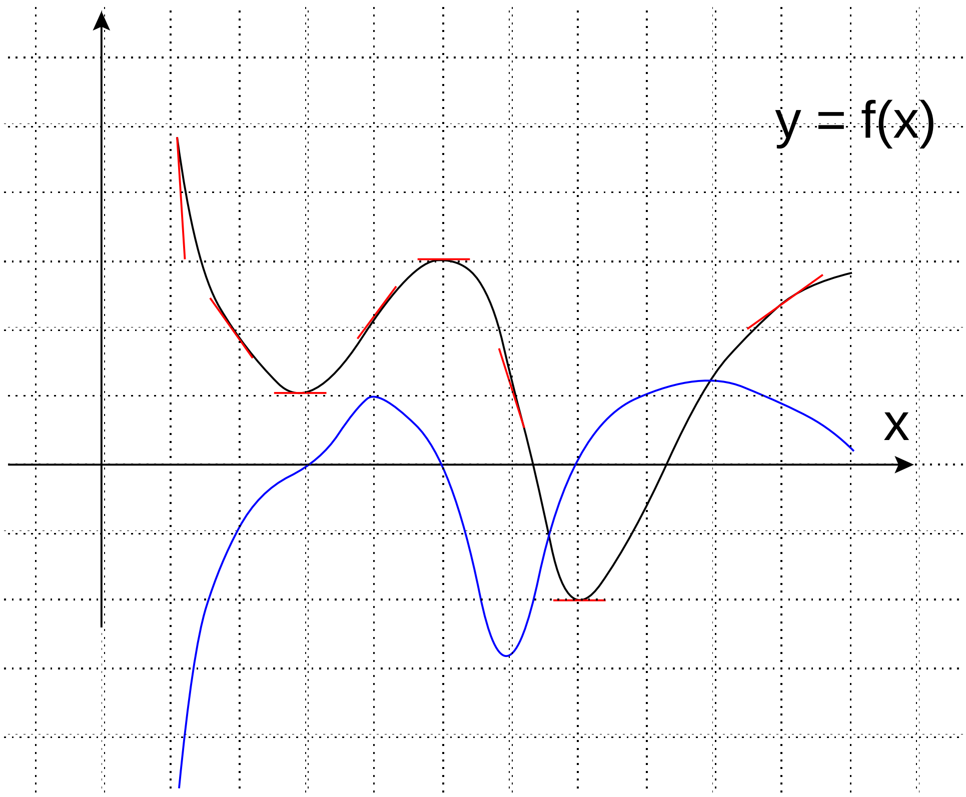
$$\frac{dy}{dt} = \frac{2\sqrt{(3+t)^2 - 4} - 2(3+t)\frac{2(3+t)}{2\sqrt{(3+t)^2-4}}}{(3+t)^2 - 4}$$

$$= \frac{-8}{[(3+t)^2 - 4]^{3/2}}$$

At time $t = 2$ s, the point P is dropping at a rate of

$$\frac{dy}{dt} = \frac{-8}{[(3+2)^2 - 4]^{3/2}} = -\frac{8}{21\sqrt{21}} \quad \text{(m/s)}$$

14. ([*10 pts*]) The graph of a function f is shown below. Draw (on the figure) the rough shape of the graph of the derivative function f'.



(A rough sketck of f' in blue.)

15. ([*10 pts*]) Let $f(x) = \sqrt{|x|}\tan|x|^{\frac{3}{2}}$. Determine if $f'(0)$ exists or not. Justify your answer.

$$f'(0) = \lim_{h\to 0} \frac{\sqrt{|h|}\tan|h|^{3/2} - 0}{h}$$

Note that

$$\lim_{h\to 0^+} \frac{\sqrt{|h|}\tan|h|^{3/2}}{h} = \lim_{h\to 0^+} \frac{\tan|h|^{3/2}}{h^{1/2}} = \lim_{h\to 0^+} h\frac{\tan h^{3/2}}{h^{3/2}}$$

$$= \lim_{h\to 0^+} h \cdot \lim_{h\to 0^+} \frac{\tan h^{3/2}}{h^{3/2}} = 0 \cdot 1 = 0$$

and

$$\lim_{h\to 0^-} \frac{\sqrt{|h|}\tan|h|^{3/2}}{h} = \lim_{h\to 0^-} \frac{\tan(-h)^{3/2}}{-(-h)^{1/2}} = \lim_{h\to 0^-} (-h)\frac{\tan(-h)^{3/2}}{(-h)^{3/2}}$$

$$= \lim_{h\to 0^-} (-h) \cdot \lim_{h\to 0^+} \frac{\tan(-h)^{3/2}}{(-h)^{3/2}} = 0 \cdot 1 = 0$$

Since

$$\lim_{h\to 0^-} \frac{\sqrt{|h|}\tan|h|^{3/2}}{h} = 0 = \lim_{h\to 0^+} \frac{\sqrt{|h|}\tan|h|^{3/2}}{h}$$

$f'(0)$ exists.

In fact, $f'(0) = 0$.